Estimates of Libor's But-for and Libor Suppression

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1 Alvarez Bio

I am the Saieh Family Professor of the Kenneth C. Griffin Economics Department of the University of Chicago. I am an Econometric Society Fellow, an Economic Theory Fellow, a Research Associate of the National Bureau of Economic Research, and a member of the American Academy of Arts & Sciences. I have served as editor of the Journal of Political Economy, associate editor at other journals, and referee for all the top economic journals. I have received grants from the National Science Foundation, the Goldman Sachs Global Market Institute (as fellow), the European Research Council, the Fondation Banque de France, the Alfred P. Sloan Foundation (as fellow), and the Tinker Foundation. I have been a Visiting Scholar at the EIEF (Enaudi Institute of Economics and Finance), and at the Research Department of the European Central Bank (as the Wim Duisenberg Fellow); a consultant at the Research Departments of the Federal Reserve of Minneapolis, the Federal Reserve Bank of Chicago, and the Federal Reserve Bank of Philadelphia; and a Visiting Researcher advising the Board of Directors of the Argentine Central Bank. My main area of research interest is macroeconomics, including asset pricing and monetary economics as subareas of specialization. Within these areas I have worked on the effects of monetary policy, the liquidity effect on interest rates, and the determination of risk premium on asset prices. My previous academic position was in the Finance Department of the Wharton School of the University of Pennsylvania. My main teaching responsibilities at the University of Chicago are the first core macroeconomics class at the doctoral level, advance topics on monetary economics at the doctoral level, and Speculative Markets at the undergraduate level. My teaching experience includes short doctoral and post doctoral courses taught at the Toulouse School of Economics, the Cowles's Foundation for Research in Economics at Yale University, the International Monetary Fund, the House of Finance at Goethe University, and the Swiss National Doctoral Program.

2 Summary

I have been asked the following questions in connection with the development of the plan of allocation. First, if there is statistical evidence of suppression on Libor Rates during the class period described below. Second, in the case of finding evidence consistent with suppression of Libor rates, to produce daily values of suppression.

The data I have collected and analyzed is consistent with suppression of Libor rates during the class period. I have used that data to produce daily estimates of suppression of Libor rates during the class period.

This document explains the procedure to calculate daily estimates of suppression for Libor rates at different tenors during the class period. The suppression estimates are based on the difference between a daily average of three Eurodollar rates and the corresponding Libor rates.

I use a three step procedure to construct daily estimates of suppression. First I use an econometric model to estimate the statistical relationships that Libor rates have had with other similarly defined Eurodollar rates using data from the outside class period. I refer to "outside class period" as the combination of two intervals of time, one pre-class and one post-class. The second step uses the econometric models estimated in the first step, and applies them to the values of Eurodollar rates during the class period to calculate a But For Libor rate for each tenor. The third step constructs the estimated raw suppression estimates as the difference between the But For Libor rates and the actual Libor rates during the class period. Finally a three day backward moving average is applied to the raw suppression estimates. This procedure yields estimates of percentage points of suppression on Libor rates during the class period relative to the levels prevailing outside the class period.[1]

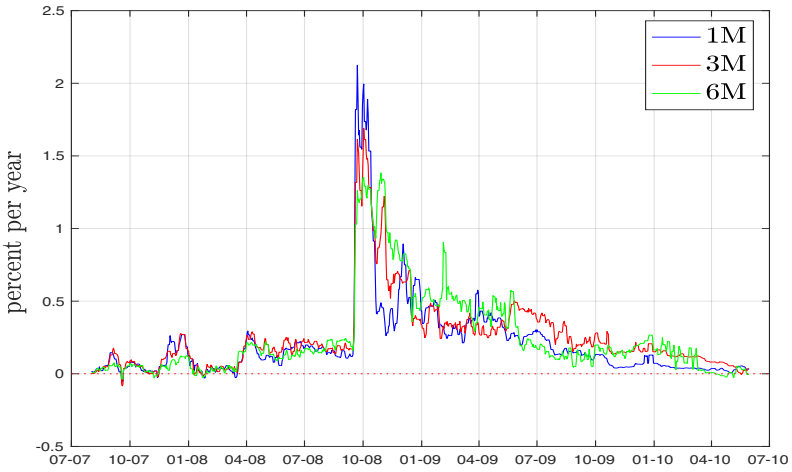
The main component of the econometric model is a weighted average of three Eurodollar rates which each measure a type of borrowing similar to the one underlying the definition of Libor rates. One of the Eurodollar rates is constructed by the Federal Reserve Bank of Saint Louis, based on information gathered by ICAP, a London based broker. The other two Eurodollar rates are constructed by Bloomberg, based on indicative quotes submitted to Bloomberg by top financial institutions. In addition to the weighted average of Eurodollar rates, the statistical model includes a constant adjustment to its level (the model's "intercept"), and a linear trend –an adjustment that changes linearly with time. The parameters for the econometric model, i.e. the weights, the value of the constant, and the coefficient for the linear trend, use data outside the class period to predict Libor rates. This estimation is performed separately for each tenor, i.e. for 1 month, 3 month, and 6 month. As shown below, my econometric model provides reliable estimates of Libor rates outside of the class period. In Figure 1 (below) I plot the estimated suppression for each tenor during the class period. This figure shows that during the class period the estimated suppression had an average value of at least 20 basis points for each of the three tenors of interest.

The rest of this report explains the nature of the Eurodollar rates used, the specification of the statistical model, the estimation of the parameters of the econometric model outside

[1] I use the term "suppression" throughout this report to refer to the differences between estimates of class period Libor rates (based on my econometric analysis of data outside the class period) and actual class period Libor rates in connection with the development of the plan of allocation. I understand that the trier of fact will make the determination whether these differences constitute an illegal suppression, as Plaintiffs have alleged.

the class period, the construction of the But For and the raw estimates of suppression, and the use of a three day backward moving average.

Figure 1: Estimated Suppression by Tenor



Note: 3 (business) days backwards moving average of raw estimates.

3 Eurodollar Rates

I use the following three daily Eurodollar rates as the predictors for the Libor rate in the statistical model. I discuss the nature of each rate here.

Eurodollar Rate from FRED. The first one is the Eurodollar Deposit Rate (London) for each tenor produced by the Federal Reserve Bank of Saint Louis. [2] I refer to these rates as Eurodollar FRED rates, or FRED rates for short. These Eurodollar rates are themselves based on the information provided by ICAP and reported by Bloomberg. ICAP is one of the largest London-based brokers of, among other products, Dollar denominated unsecured deposits. Its clients are mostly banks. The ICAP rates are determined by their brokers, reflecting the information that they see, as a consequence of their activities as brokers, to represent the rates that top tier banks (or other institutions) will obtain for unsecured dollar borrowing.[3] These rates are meant to be representative of deposits to be placed by these institutions in banking centers in London, Caribbean, or elsewhere outside of the United States. These rates are neither averages of transactions, nor surveys of participants.

Eurodollar Rates from Bloomberg. The second source I use for Eurodollar rates are Bid and Ask best market composite rates for Dollar unsecured lending, measured at the end of the London business day, as produced by Bloomberg. These best market composite rates use Bid and Ask submissions of indicative quotes by institutions deemed to be open at or immediately before the end of London business time. I refer to them as Bid and Ask Eurodollar Bloomberg rates, or Bloomberg rates for short.

An indicative quote is a rate (either a bid or an ask) submitted by participants (mostly by banks, but also by brokers) to Bloomberg for unsecured dollar lending (or borrowing), and which are available in real time to Bloomberg clients. These quotes allow market participants both to have an idea of where the market is at a given time, and also to be able to contact the submitter of the rate to negotiate a trade.[4] Using these individual submitter quotes, the best market composite rate is defined as follows: A subset of the participants submitting bid and

[2] The exact source is the Board of Governors of the Federal Reserve System (US), 1-Month Eurodollar Deposit Rate (London) [DED1], 3-Month Eurodollar Deposit Rate (London) [DED3], and 6-Month Eurodollar Deposit Rate (London) [DED6], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/DED1 or /DED3 or / DED6 respectively.

[3] For this purpose, top tier institutions are a subset of prime banks that excludes those that appear to be trading as an outlier for credit reasons on a given day. Prime banks are generally those banks with credit rating A1 or P1.

[4] The bid and ask indicative quotes differ from the bid and ask of a market maker in that to transact with the institutions that submitted the indicative quotes, one has to first contact the submitter, after which rates may be renegotiated, depending on the size of the proposed transactions and other considerations.

ask quotes are denoted by Bloomberg as privileged based on the quality and consistency of their data, as well as for their consensus with the market. Bloomberg computes a best bid and best ask at each moment of time using only quotes from the privileged submitters. Bloomberg uses only "recent" quotes to compute the best bid and best ask, i.e., quotes that are less than five minutes old. Additionally, Bloomberg requires privileged institutions be "open" (i.e., during regular business hours) at the time at which quotes are submitted in order to be considered in the composite bid and ask calculation. The best bid is the highest bid quote among privileged participants from institutions deemed to be open and submitted within the prior five minutes. The best ask is the lowest ask quote among privileged participants from institutions deemed to be open and submitted within the prior five minutes. I have access to best bid and best ask rates over the entire day from 2008 to the end of 2016, and access to best bid and best ask rates at the time of market closing for London from 1996 to the end of 2016.[5]

Eurodollar rates as proxies for Libor rates. These Eurodollar rates correspond to borrowing conditions that are similar to those underlying the definition of Libor rates. Libor is defined as the ask rate at which top tier banks (i.e. those on the British Bankers' Association panel) could obtain unsecured dollar funds in the London interbank market. The FRED and Bloomberg Eurodollar rates are proxies for *Eurodollar* deposits in the following sense. FRED Eurodollar rates correspond to the London interbank market because of their reliance on ICAP, a London based broker, which strives to represent the range of rates for deposits in London, the Caribbean, and outside the US. In the case of the Bloomberg best market rates, they correspond to London interbank market rates because they are measured at the end of London business day, submitted by institutions that are deemed to be operating in the London market. Furthermore, FRED and Bloomberg Eurodollar rates are proxies for interbank borrowing because, while they do not exclusively reflect activity between banks, banks are among the main participants. Finally, both FRED and Bloomberg Eurodollar rates correspond to unsecured deposits among institutions that are highly rated, and hence, roughly comparable to those in the BBA panel. Indeed the *levels* of the Eurodollar FRED and Eurodollar Bloomberg Ask rates are similar to those of the Libor rates outside the class period. Using spreads between Libor and other Eurodollar rates, the model accounts for daily common macroeconomic factors.

As mentioned above, consistent with the description of these Eurodollar rates as valid proxies for Libor rates, outside the class period the statistical model gives a very close pre-

[5]The end of day London time rates are referred to by Bloomberg as CMPL USD rates. The Bloomberg names of the 1M, 3M, 6M, and 9M (`Bid` and `Ask`) end of London rates are: USDRA CMPL, USDRC CMPL, USDRF CMPL, and USDRI CMPL (`PX_BID` and `PX_ASK`) respectively.

diction of Libor rates. Figure 3, Figure 4, and Figure 5 (below) plot the difference between each of these rates and the Libor rates for each tenor during the entire period. From these figures, it is clear that the difference between these Eurodollar rates and Libor rates are much smaller outside the class period than during the class period. Figure 6, Figure 7, and Figure 8 (below) plot the difference between the But For Libor rates (i.e. those estimated by the econometric model) for each relevant tenor and the corresponding Libor rates. These figures show that the fit of the statistical model outside the class period is very good, i.e. the differences between the predicted and actual Libor rates are typically very small. I measure the fit using the standard deviation of the error in the regression, denoted by σ_e below. The interpretation of this statistic is the typical discrepancy between the prediction of the model and the actual Libor rate outside the class period. The standard deviation of these errors are 3.8, 3.4 and 4.2 *basis points* for 1M, 3M and 6M tenors, respectively (see Table 1). To see that these errors are very small, one can compare their typical magnitude with the standard deviation of Libor rates themselves, which measure the typical difference of these rates relative to their average value outside the class period. The Libor rates are plotted in Figure 10. The standard deviation of Libor rates outside the class period are about 60 times higher than the standard deviations of the prediction errors. The standard deviation of the Libor rates are 2.36, 2.34, and 2.29 *percentage points* for 1M, 3M and 6M tenors respectively –see also Table 3 below for more details. Thus, the typical prediction error of the statistical model is much smaller than the typical variation of Libor rates.

As described above these eurodollar rates have the same properties that define the Libor rates: they correspond to unsecured dollar borrowing rates by high quality financial institutions in the London interbank market. Thus, macroeconomic factors as well as market wide factors in the unsecured interbank market should impact them in the same way as they impact Libor rates. Finally, I have chosen a pre-class period that goes back in time far enough to include a period of turmoil in the interbank market. A commonly used measure for turmoil in the interbank market is the TED spread, defined as the the difference between the Libor and Treasury rates of the same tenor. Figure 9 plots the TED spread during the relevant time period for the 3M tenor. Our pre-class period includes the end of the 90s and early 00s, which had elevated TED spreads. The other period of time with very large TED spreads is the class period, which overlaps with the 2007-2008 financial crisis. Thus, the inclusion of the early period is important because I use the relationship of Libor and its proxies outside the class period to forecast what the Libor rates should have been during the class period, in the absence of any manipulation. Hence, using a pre-class period with elevated TED spreads to estimate the econometric model enhances its reliability as a counterfactual during the class period, which also has elevated TED spreads.

The "class" and "outside" periods. The "class period" begins at Aug-1-2007 and ends at May-31-2010. The "outside period" consists of two intervals of time, a pre-class period and a post-class period. The pre-class period begins at September-25-1996 and ends at July-31-2007. The post-class period begins at June-1-2010 and ends at September-25-2016.

4 Econometric Model

To discuss the exact specification of the econometric model I first establish some notation. In this discussion, I concentrate on a given tenor, say 3 months, or 3M for short. I follow the same methodology for the other tenors. As discussed above, I estimate the model for each tenor separately. I denote consecutive calendar dates by t. I denote by L_t the daily value of the 3M Libor rate, F_t the daily value of the Eurodollar FRED Rate rate at t, BA_t the Eurodollar Bloomberg Ask rate at date t, and BB_t the Eurodollar Bloomberg Bid rate at date t.

The baseline model is:

$$L_t = \beta_0 + \beta_1 F_t + \beta_2 BA_t + \beta_3 BB_t + \beta_4 T_t + e_t \text{ for } t \text{ outside the class period} \quad (1)$$

The variable T_t is a linear *trend*, i.e. it is proportional to calendar time t. Note that there are several dates for which either the Libor rate, or some of the Eurodollar rates which I use as regressor (i.e, variables used to predict the Libor rate) are not available. Almost all of these dates are on week-ends and London banking holidays, as well as days at which the Bloomberg Eurodollar rates were not produced by Bloomberg. I excluded these dates from the regression. Moreover I restrict the coefficients so that

$$\beta_1 + \beta_2 + \beta_3 = 1 \quad (2)$$

This restriction is imposed because these rates have large persistent common movements. See Figure 10 for the levels, which shows substantial persistent variation, and Figure 3, Figure 4, and Figure 5 for the spreads of Libor Rates with Eurodollar rates, which show substantial mean reversion. This specification simply means that the Libor But For rate is estimated by a weighted average of the Eurodollar Fred, Eurodollar Bloomberg Ask, and Eurodollar Bloomberg Bid, including a constant difference and a linear trend.

The restriction in Equation 2 is imposed by running OLS regression in *spreads* as follows:

$$L_t - F_t = \alpha_0 + \alpha_1 (BA_t - F_t) + \alpha_2 (BA_t - BB_t) + \alpha_3 T_t + e_t \text{ for } t \text{ outside the class period} \quad (3)$$

Using the estimated values I can write Equation 3 as:

$$L_t = \alpha_0 + (1-\alpha_1)F_t + (\alpha_1+\alpha_2)BA_t - \alpha_2 BB_t + \alpha_3 T_t + e_t \text{ for } t \text{ outside the class period}$$

so that I obtain the values of $\beta's$ from:

$$\beta_0 = \alpha_0,\ \beta_1 = 1-\alpha_1,\ \beta_2 = \alpha_1+\alpha_2,\ \beta_3 = -\alpha_2,\ \beta_4 = \alpha_3 \tag{4}$$

The model in Equation 3 can also be interpreted directly as hypothesizing that the spread between Libor and Eurodollar FRED is explained by the spread between Eurodollar FRED and Eurodollar Bloomberg Ask, and by the Bid-Ask spread using Eurodollar Bloomberg rates.

I estimate the parameters α of the model given by Equation 3 using OLS. I obtain estimates for the parameters β using Equation 4 and the estimated values for α.[6] The model is statistically reliable in terms of the statistical significance of its coefficients, using Newey-West standard errors at conventional statistical levels.

Linear Trend. The main specification contains a linear trend. The trend is included because, as can be seen in Figure 3, Figure 4, and Figure 5, at the beginning of the class period there is a tendency to have lower values of the Eurodollar rates relative to Libor than at the end of it. Indeed, the estimated value of the coefficient for the trend, α_3, is negative.[7] Since the coefficient is very small, it has a small effect on the estimates of suppression, as well as a relatively small effect on improving the fit of the model.

5 But For and Suppression Estimates

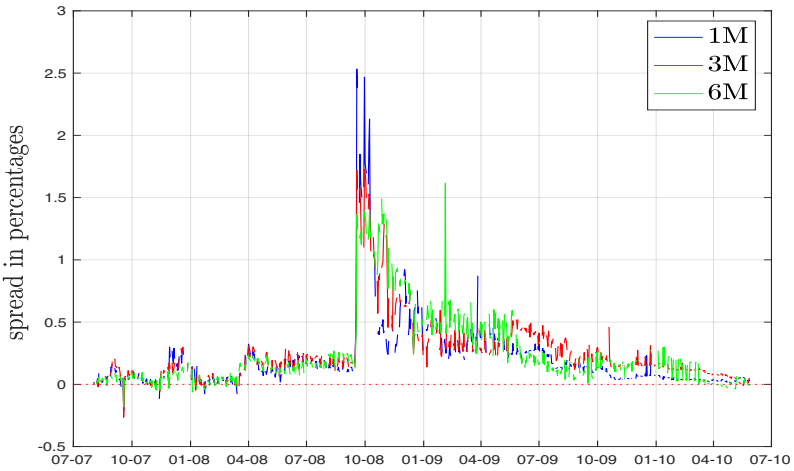
The estimated coefficients for the spread model of Equation 3 are provided in Table 1. In that table I express the coefficients in terms of the Equation 1, using the linear Equations 4 and the estimated coefficient from Equation 3. I produce daily estimates of the But For Libor rate during the class period using the values of the Eurodollar Rates during that period. The raw daily suppression estimates are equal to the But For Libor rate estimates minus their corresponding actual Libor rates –see Equation 5 below. The raw suppression estimates for

[6] Equivalently, one can estimate the parameters β using Equation 1 and imposing the constraint in Equation 2.

[7] I use the following linear transformation of the linear trend to aid in the interpretation of its coefficient. I divide t by 365 so that its coefficients has the units of a daily trend. Thus $T_{t+1} - T_t = 1/365$. I also subtract a constant, so that the mean of trend T_t is zero outside the class period. This mean is taken relative to the dates outside the class period where none of the variables are missing.

tenors 1M, 3M, 6M are plotted in Figure 2.

Figure 2: Raw Estimated Suppression by Tenor



All the coefficients have economically sensible signs and magnitudes. The estimated values of β_1, β_2, and β_3 are all positive, so that the estimate for the But For Libor is a weighted average of the Eurodollar rates. The constant is positive, which partially reflects the fact that the Bloomberg Eurodollar Bid rate is, on average, lower than Libor rate. The coefficient for the linear trend is negative and small in absolute value.

The daily raw estimates of suppression for a given tenor are thus given by:

$$\text{Raw Suppression}_t = \beta_0 + \beta_1 F_t + \beta_2 BA_t + \beta_3 BB_t + \beta_4 T_t - L_t \tag{5}$$

for dates t inside the class period.

Table 2 presents summary statistics of the raw suppression estimates during the class period across different tenors. The average suppression during the class period is above 20 basis point for all of these tenors. As can be seen from the table, as well as from the figures displaying estimated suppression, the estimated suppression is particularly high right after Lehman bankruptcy, beginning in mid September 2008. The extreme values during the period after the Lehman bankruptcy are also reflected in the larger value of its mean relative to its median.

One-sided moving average of suppression. For the purpose of estimating the suppression at the time of the reset of the coupon rate of variable rate bonds, I use a one-sided moving average of the daily raw estimates of suppression by tenor. The one-sided moving

Table 1: Regression coefficients ($\beta's$) based on spread model

	1M	3M	6M
Constant (β_0)	0.03434	0.03303	0.02786
FRED (β_1)	0.7474	0.71795	0.63594
BBG ask (β_2)	0.08565	0.15482	0.22695
BBG bid (β_3)	0.16695	0.12723	0.13711
Trend (β_4)	-0.00927	-0.00838	-0.00843
σ_e	0.03817	0.03425	0.04238
R^2	0.74808	0.75647	0.65188
Sample Size	4201	4201	4207

Note: The estimates of β are constructed using Equation 3, and the estimates of α. The estimates of α are obtained fitting OLS to Equation 3. For the estimates I pool daily data during the pre- and post-class periods. The data excludes week-ends and London holidays, as well as dates where at least one of the regressors is not available. The R^2 and σ_e correspond to the spread model of Equation 3.

Table 2: Statistics on Suppression Estimates during class period

	1M	3M	6M
Average	0.2100	0.2545	0.2496
Median	0.1324	0.1771	0.1535
Standard Deviation	0.1324	0.2741	0.3060
Number of days w/positive suppression	674	672	639
Number of days w/negative suppression	12	21	42
Number of days without rates	347	340	352
Total number of days	1033	1033	1033

Note: Based on raw estimates of suppression. All statistics are in percent per year.

average during a reset day is the average of the raw estimates of daily suppression over the reset day and the previous two business days. If a reset day is not a business day, or if, for some reason, the estimates are unavailable –sometimes because some of the Eurodollar rates were not published that day– I use the consecutive previous three business days.[8] I use this moving average for two reasons. First and foremost, for most variable rate bonds, the actual rate used for payments is not the Libor rate published on the reset day, but instead it is the Libor rate published a few business days prior, most commonly two days. Second, the Bloomberg Eurodollar rates are more volatile than the Libor rates that my model is predicting– and thus the raw suppression rates reflect, to some extent, the volatility of the Bloomberg Eurodollar rates. A moving average is a simple and well-accepted way to reduce the inherited volatility.[9] The one-sided moving averages of the raw suppression can be seen in Figure 1 for tenors 1M, 3M and 6M.

Other maturities. I concentrate on estimating Libor suppression for the 1M, 3M, and 6M tenors because almost all the Libor variable rates bonds outstanding during the class

[8]For the beginning of the class period I use either the two day or one day moving average.

[9]The OLS estimates also take the extra volatility into account on the determination of the coefficients β_1, β and β_3.

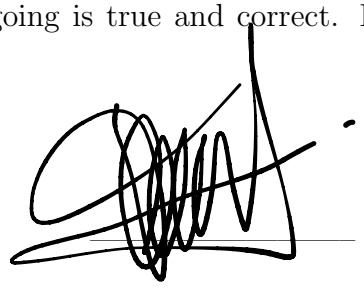
period set their coupon payments using these three tenors. I extend the estimates of Libor suppression to other tenors as follows. Since some of the benchmark Eurodollar rates I used for the estimation of suppression for the 1M, 3M and 6M tenors are not available for all tenors, I use extrapolation and interpolation methods for those cases. I obtain estimates of the suppression rates for the following tenors: 1W, (one week), 2M (two months), 4M (four months), 5M (five months), 8M (eight months), 9M (nine months), and 12M (twelve months). They are displayed in Figure 11.

Suppression Estimates. The daily suppression estimates for tenors 1W, 1M, 2M, 3M, 4M, 5M, 6M, 8M, 9M, and 12M can be found in the attached spreadsheet.

6 Conclusion

It is my expert opinion that the data analyzed is consistent with suppression of the Libor rate during the class period. It is also my expert opinion that the econometric model I used provides reliable estimates of the but-for-Libor rate during the class period.

I declare under penalty of perjury that the foregoing is true and correct. Executed on August 21st, 2019.

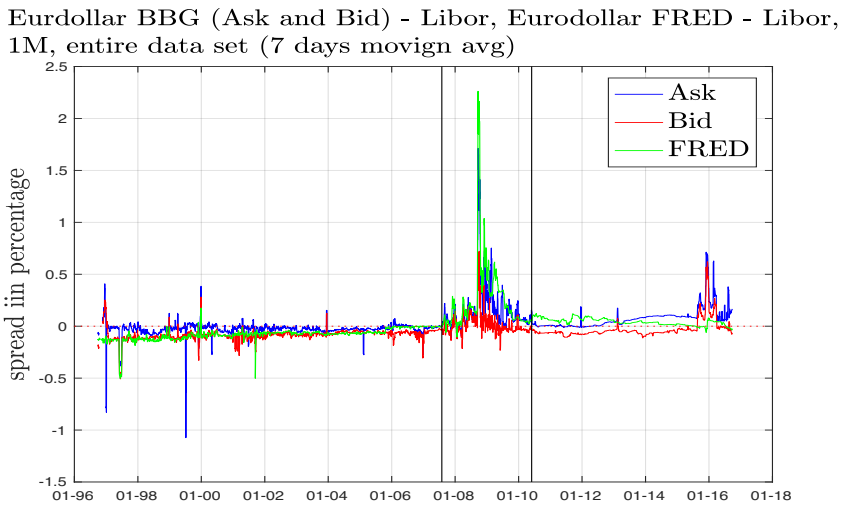


Fernando Alvarez

7 Appendix: Figures and Tables

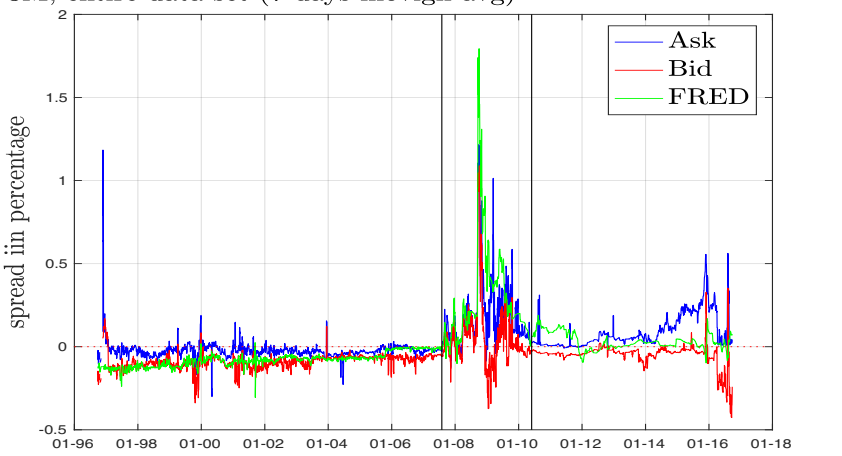
This section contains figures and tables referred to in the main body.

Figure 3: Eurodollar Libor Spread, One month Tenor



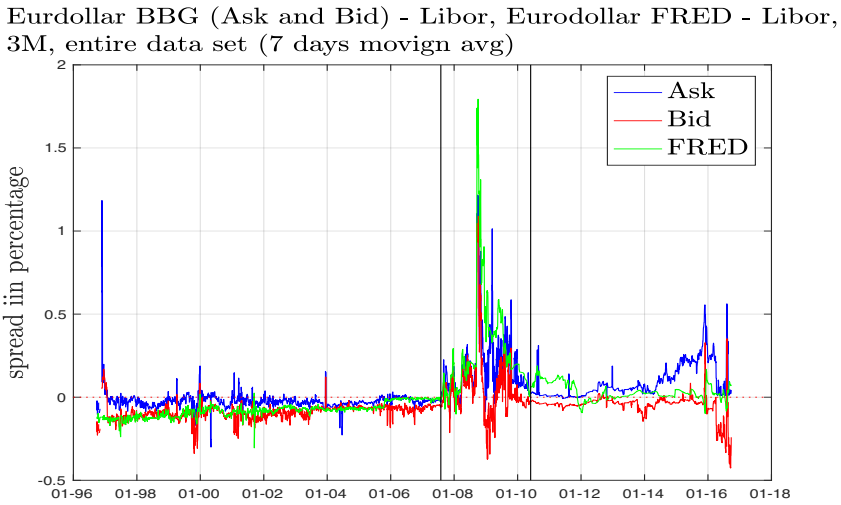
Note: 7 days center moving average of data

Figure 4: Eurodollar Libor Spread, Three month Tenor



Note: 7 days center moving average of data

Figure 5: Eurodollar Libor Spread, Six month Tenor



Note: 7 days center moving average of data

Figure 6: But For (statistical model) - Libor Rate, One month Tenor

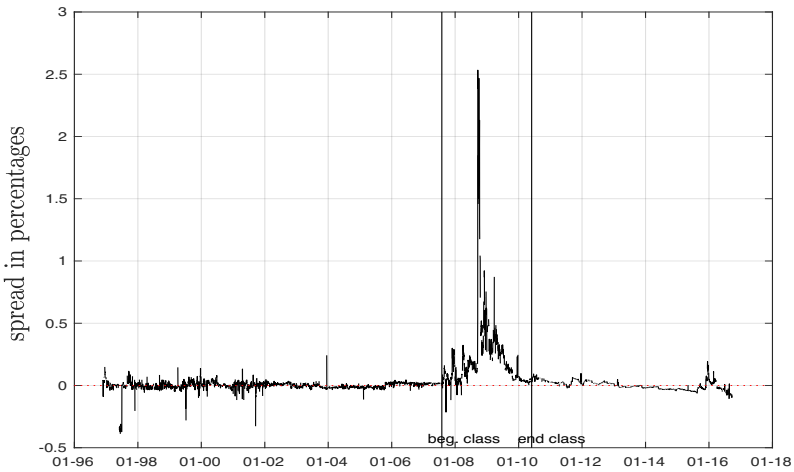


Figure 7: But For (statistical model) - Libor Rate, Three month Tenor

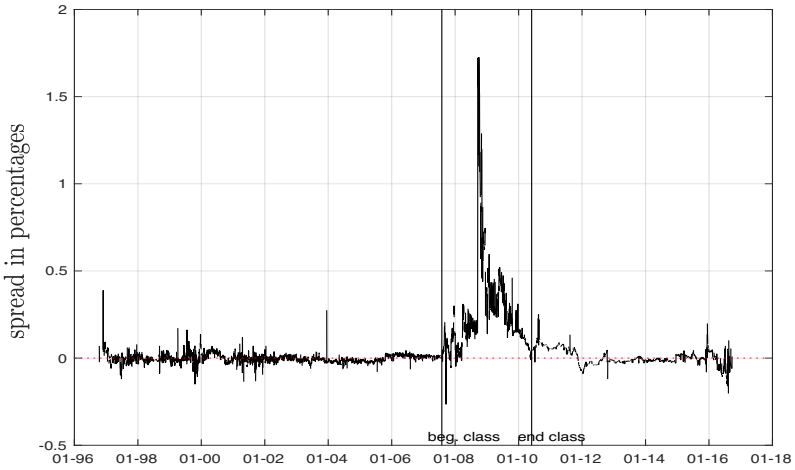


Figure 8: But For (statistical model) - Libor Rate, Six month Tenor

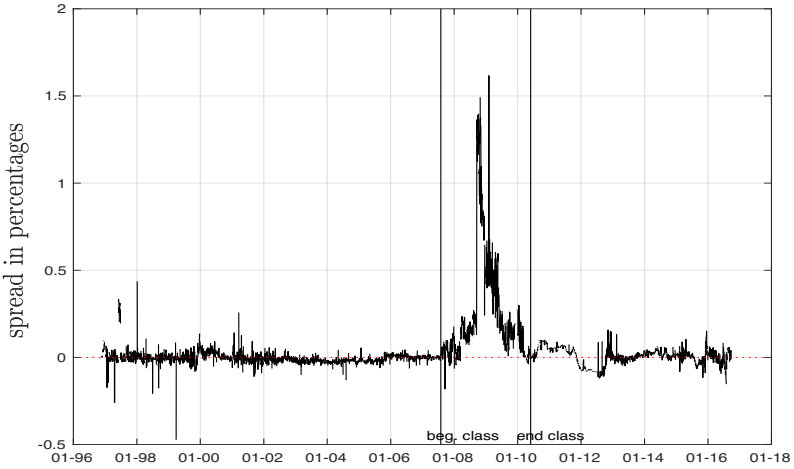
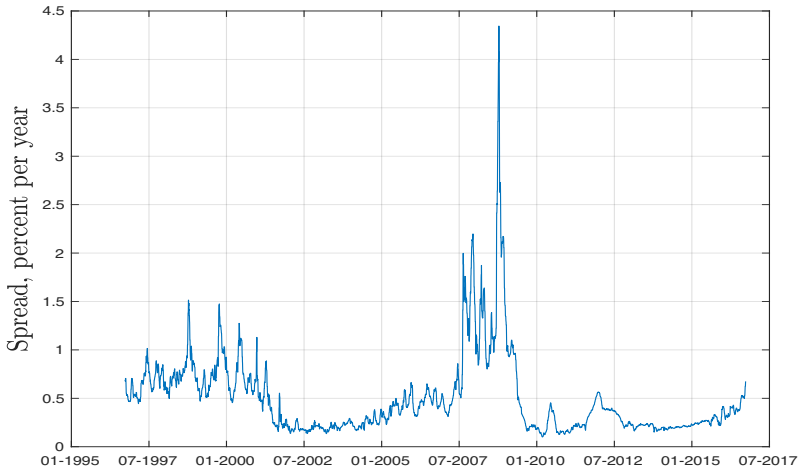


Figure 9: Daily Ted spread = Libor 3M - Treasury 3M



Note: 7 days center moving average of data

Figure 10: Daily Libor Rates for different tenors

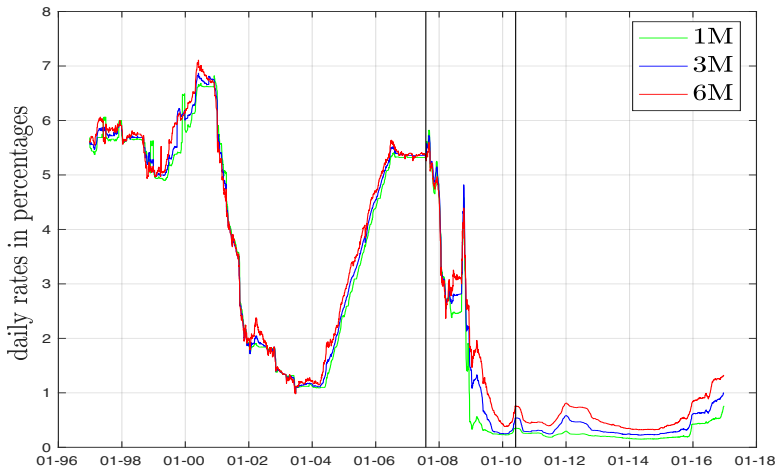


Table 3: Standard Deviation of Libor Rates Pre-Class, Class, and Post-Class

	1M	3M	6M
Pre class	1.866	1.876	1.868
Post class	0.113	0.180	0.251
Outside class	2.357	2.339	2.295
During class	1.819	1.753	1.592
Entire Period	2.302	2.275	2.216

Note: Standard deviations of Rates measured in percentage per year. Total number days with quotes 5053, days during class period 715, days outside class period 4338, days during pre class period 2676, and days during post class period 1662.

Figure 11: Libor Suppression, other tenors